

Assignment 5 Solutions

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Introduction to Mathematical Thinking

Answer 1

- (a) $(\exists x \in \mathbb{N})[x^3 = 27]$
- (b) $(\exists x \in \mathbb{N})[x > 1000000]$
- (c) $(\exists a \in \mathbb{N})(\exists b \in \mathbb{N})[a > 1 \wedge b > 1 \wedge n = ab]$

Answer 2

- (a) $(\forall x \in \mathbb{N})[x^3 \neq 28]$
- (b) $(\forall x \in \mathbb{N})[0 < x]$
- (c) $(\forall a \in \mathbb{N})(\forall b \in \mathbb{N})[(n = ab) \Rightarrow (a = 1 \vee b = 1)]$

Answer 3 Let P denote the set of all people.

- $LUV(x, y) := x \text{ loves } y$
- $T(x) := x \text{ is tall}$
- $S(x) := x \text{ is short}$
- $H(x) := x \text{ is at home}$
- $W(x) := x \text{ is a woman}$
- $M(x) := x \text{ is a man}$
- $L(x) := x \text{ leaves}$
- $C(x) := x \text{ comes}$

- (a) $(\forall x \in P)(\exists y \in P)[LUV(x, y)]$
- (b) $(\forall x \in P)[T(x) \vee S(x)]$
- (c) $(\forall x \in P)[T(x)] \vee (\forall x \in P)[S(x)]$
- (d) $(\forall x \in P)[\neg H(x)]$
- (e) $C(\text{John}) \Rightarrow (\forall x \in P)[W(x) \Rightarrow L(x)]$
- (f) $(\exists x \in P)[M(x) \wedge C(x)] \Rightarrow (\forall x \in P)[W(x) \Rightarrow L(x)]$

Answer 4

- (a) $(\forall a \in \mathbb{R})(\exists x \in \mathbb{R})[x^2 + a = 0]$
- (b) $(\forall a \in \mathbb{R})[a < 0 \Rightarrow (\exists x \in \mathbb{R})[x^2 + a = 0]]$
- (c) $(\forall x \in \mathbb{R})(\exists p \in \mathbb{N})(\exists q \in \mathbb{N}) \left[x = \frac{p}{q} \vee x = -\frac{p}{q} \vee x = 0 \right]$
- (d) $(\exists x \in \mathbb{R})(\forall p \in \mathbb{N})(\forall q \in \mathbb{N}) \left[x \neq \frac{p}{q} \wedge x \neq -\frac{p}{q} \wedge x \neq 0 \right]$
- (e) $(\forall y \in \mathbb{R})(\exists x \in \mathbb{R}) \left[(x > y) \wedge (\forall p \in \mathbb{N})(\forall q \in \mathbb{N})(x \neq \frac{p}{q} \wedge x \neq -\frac{p}{q} \wedge x \neq 0) \right].$

Answer 5 Let C be the set of all cars, $D(x) := x$ is domestic, and $M(x) := x$ is badly made.

- (a) $(\forall x \in C)[D(x) \Rightarrow M(x)]$
- (b) $(\forall x \in C)[\neg D(x) \Rightarrow M(x)]$
- (c) $(\forall x \in C)[M(x) \Rightarrow D(x)]$
- (d) $(\exists x \in C)[D(x) \wedge \neg M(x)]$
- (e) $(\exists x \in C)[\neg D(x) \wedge M(x)]$

Answer 6

$$(\forall x \in \mathbb{R})(\forall y \in \mathbb{R}) [(x < y) \Rightarrow (\exists z \in \mathbb{R}) (Q(z) \wedge (x < z < y))]$$

Answer 7 Let $F(p, t)$ denote "You can fool person p at time t ".

$$(\exists t)(\forall p)[F(p, t)] \wedge (\exists p)(\forall t)[F(p, t)] \wedge \neg[\forall p \forall t F(p, t)]$$

Answer 8

$$\exists x \forall t A(x, t)$$

Answer 9

$$\forall t \exists x A(x, t)$$