Assignment 2 Solutions

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Introduction to Mathematical Thinking

Answer 1 (b) $7 \leq p < 12$

(c) $5 < x < 7$

(d) $x < 4$

(e) $-3 < y < 3$

(f) $x = 0$

Answer 2 (b) p is equal to or greater than 7, but less than 12.

(c) x is greater than 5, but less than 7.

(d) x is less than 4.

(e) y is greater than -3, but less than 3.

(f) x is equal to 0.

Answer 3 I will show that every single element of $\{\varphi_1, \varphi_2, \ldots, \varphi_n\}$ is true. If one or more elements in this set are false, the whole conjunction will become false.

Answer 4 I will show that just one single element of $\{\varphi_1, \varphi_2, \ldots, \varphi_n\}$ is false. Then it will make the whole conjunction false.

Answer 5 (a) $\pi > 3$

(b) $\{x \in \mathbb{R} \mid x \neq 0\}$

(c) $x \geq 0$

(d) $x \geq 0$

(e) $x^2 > 9$

Note: Comparing algebraic symbols such as $x > 0$ or $x < 10$ implies that x is a real number, but the not-equal sign $\neq$ does not. Therefore, I indicate the range of x in (b).

Answer 6 (a) π is greater than 3.

(b) x is every real number such that x isn't 0.

(c) x is equal to or greater than 0.

(d) x is equal to or greater than 0.

(e) x^2 is greater than 9.

Answer 7 I will show that just one single element of $\{\varphi_1, \varphi_2, \ldots, \varphi_n\}$ is true.

Answer 8 I will show that every single element of $\{\varphi_1, \varphi_2, \ldots, \varphi_n\}$ is false.

Answer 9 (a) $\pi \leq 3.2$

(b) $x \geq 0$

(c) $(x = 0) \vee (x \notin \mathbb{R})$

(d) $x \neq 1$

(e) ψ

Answer 10 (a) π is equal to or less than 3.2.

(b) x is equal to or greater than 0.

(c) x is equal to 0, or x is a complex number that is not a real number.

(d) x is not equal to 1.

(e) ψ is true.

Answer 11 (a) $D \wedge Y$. It's the meaning of the "and" operator, which means both elements are true.

(b) $\neg Y \wedge T \wedge D$. Despite and but can tell us that we have a conjunction.

(c) $\neg(D \wedge Y)$. This is a negation of "Dollar and Yuan are strong at the same time." If Dollar and Yuan are both weak at the same time, or one of the two is strong, the statement will be true.

(d) $T \wedge \neg D \wedge \neg Y$. I assume the writer's intention is that a new trade agreement was made, but Dollar and Yuan fell.

(e) $\neg T \wedge D \wedge Y$. Straightforward.

Answer Two to Think About 1. Of course, it's not the same. The meaning of "¬ guilty" in a mathematical sense is that the innocence of the accused person is proven. But in a trial, the defendant is "not guilty" when the prosecution fails to prove guilt within a limited amount of time. So this state of affairs may or may not be the "not" in the mathematical sense, because the defendant is "not guilty" either when innocence is proven, or when the proving process has failed.

2. The problem of expressing "not displeased" as "¬¬ pleased" in logical language is that there is no such "middle place" between them. The meaning of "I was not displeased with the movie" in everyday life is "The movie was so-so, I was neither pleased nor displeased." My solution is to introduce both "pleased" and "displeased" as statements. So it can be (¬pleased) ∧ (¬displeased), or ¬(pleased ∨ displeased).